

# Optimal Imperfection of Reactive Powder Concrete Slabs under Impact Load

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**Abstract**— Optimization of imperfect reactive powder concrete slabs under impact load is the aim of the current study, the study adopts to investigate the optimal imperfection to decrease the maximum dynamic deflection which affects negatively on structures. Optimization by using Modified Hooke-Jeevs method of imperfect slabs has been adopted in three cases according to the required variables, optimal thickness, optimal thickness-length and optimal thickness-length-imperfection. The results show that the optimal ratio of imperfect reactive powder concrete slab thickness to the one of its dimension is equal to (0.049) whereas the optimal ratio of imperfection to the thickness of slab is equal to (0.785) . The current study adopted preparing designable table which is considered informative for any future designs.

**Keywords**— Optimization, imperfect slabs, cambered slabs, impact load.

## I. INTRODUCTION

Imperfect slab is a structural technique used to gain membrane action, in which more resistant can be obtained, it is a cambered slab in between of flat and shell, Figure (1). The topology of the slab should be generated in accordance with the following relation:

$$z = c \left[ \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \right] \quad (1)$$

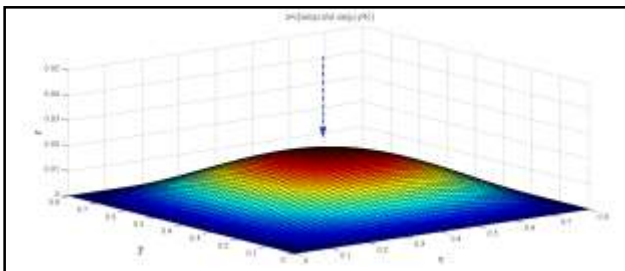


Fig. 1: Surface Drawing of Maximum Dynamic Deflection of Equation

Imperfect slab is considered very economic member where extra resistant can be obtained by using structural technique, the structural technique is represented by the mid-span camber in which membrane forces will be activated, hence, this advantage happens without significant extra materials. Imperfect reactive powder

concrete slabs can be used in threatened structures by terrorist attack, war or missile attack, such as nuclear plants, power plants, weapon industries, defense structures and security enclosures. In recent years, the design of imperfect plates has been enjoyed by special attention by utilizing the main principle of large deflection theories, in which in-plane and transverse deformation of plate were taken into consideration.

Jianqiao (1994) discussed the non-linear behavior of rectangular thin plates with initial imperfection by using finite element and boundary element method, the results show that mid-span deflection of either clamped or simply supported plate increases with the decreasing in value of imperfection. In the other hand, Shhatha et. al. performed an experimental study and submitted relations of imperfect reactive powder concrete slab under impact load, the results show that cambered reactive powder concrete slab revealed significant resistance compared with other stiffened slabs, despite that the impact force was the largest amongst all specimens, it was found that increasing the cambering (mid-span camber) of (1cm) and (2cm) causes decreasing in maximum mid-span deflection about (47%) and (65%) respectively. None of the two researchers investigated the optimal imperfection, hence, an optimization with different variables has been taken into consideration in the current study.

## II. MATHEMATICAL OPTIMIZATION METHOD

Most of the design problems and studies have several solutions and the main topic for any design or study is obtaining the best solution (optimum solution). Finding the optimum solution by classical seeking (try and error) among the variables is considered acceptable when the number of variables is few while this method becomes invalid when the number of variables is large. The development in computer increases the number of optimization methods where there are very large numbers but everyone has a limitation for its use.

Optimization is done for any case according to the purpose that is required in the field. Engineers pursue to get the best specification, cost and time and they take many technical or administration decisions to minimize the efforts or maximize the benefits (min cost, min

weight, min production time, max shear, max torque....etc.). These decisions need to be in a good sequence to reach for the aim, that concept is represented by the optimization, so many mathematical programming are produced to deal with that idea. The main important thing is how to formulate the problem in field and change it in function to be ready for using in a specific program. Finally, The restrictions that bind the problem and they cannot pass them are defined as constraints. Figure (2) shows that  $g(x)$  and  $h(x)$  are constraints for the problems  $f(x)$  where the optimum value will be one of the values of  $f(x)$  but in between of  $g(x)$  and  $h(x)$ .

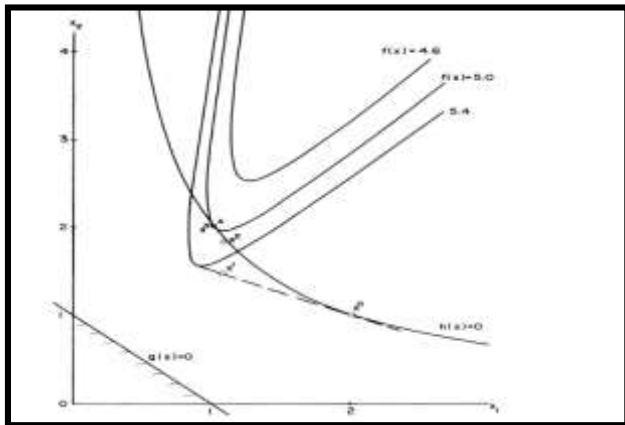


Fig. 2: Constraints for  $f(x)$  function

In (1961) Hooke and Jeeves had suggested direct-search method for optimization for an objective function without constraints, in (1984) Bunday had modified this method where the method become to be used for an objective function with constraints by suggesting that merely giving the objective function a very large value ( $10^{30}$ ). The method can be briefed in the following points:

- 1- Suggest an initial value .....Checked with constraints
- 2- Make the first exploration .....Checked every step with the constraints
- 3- Make pattern move ..... Checked with constraints
- 4- Make the second exploration..... Checked every step with the constraints
- 5- Terminate the process when the step length has been reduced to a small value.

The method is considered very suitable for the problem has large number of constraints but the method is not able to move along the constraint and converges on the first point on the constraint that it locates as the solution so searching along the initial variables has to be done to avoid that problem.

The current study adopted to make optimization for the results of Shhatha et. al. where the researcher submitted an equations as shown below:

$$c/h = 0.167 \quad (R^2 = 0.9819)$$

$$W_d = 0.0771 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1305} \quad (2)$$

$$c/h = 0.5 \quad (R^2 = 0.9842)$$

$$W_d = 0.0289 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1383} \quad (3)$$

$$c/h = 1 \quad (R^2 = 0.9878)$$

$$W_d = 0.0087 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1521} \quad (4)$$

$c$  : Magnitude of cambering at mid span

$h$  : Thickness of slab

$R^2$  : The correlation coefficient

$F_m$ : Maximum impact force at mid span deflection

$a$  : Dimension of slab

$D$  : Flexural rigidity of slab

$W_d$ : Maximum mid span deflection during the displacement-time history

Formulating the objective functions, constraints and other details will be illustrated by according to number of variables, they have been tabulated elsewhere.

### III. THEORETICAL STUDY AND RESULTS

3.1 GENERAL INVESTIGATION OF IMPERFECT SLABS: Shhatha et. al. investigated an equations about imperfect slabs under impact load but the study didn't take into consideration the optimum magnitude of imperfection. The current study submits study about the optimum imperfection that will mainly base on reducing the maximum deflection during the impact load. An overview of those equations for various cases has been clarified in Table (1).

Table.1: Maximum Dynamic Deflection for Various cases of Imperfect Slabs

Slab thickness (cm)	Imperfection (cm)	Dimensions (cm)				
		100*100	200*200	300*300	400*400	500*500
3	0.5	0.26	0.38	0.46	0.54	0.61
	1.5	0.1	0.14	0.18	0.21	0.24
	3	0.03	0.046	0.059	0.07	0.081
4	0.668	0.3	0.43	0.53	0.62	0.7
	2	0.11	0.16	0.2	0.24	0.27
	4	0.034	0.052	0.066	0.079	0.09
5	0.835	0.33	0.48	0.59	0.69	0.78
	2.5	0.12	0.18	0.23	0.27	0.3
	5	0.037	0.056	0.072	0.086	0.099

It is observed that the lower magnitude of maximum dynamic deflection happens in slabs that have minimum dimensions and thicknesses with maximum imperfection as shown in shaded cell, on the other hand, upper magnitude of maximum dynamic deflection satisfies in slabs that have maximum dimensions and thicknesses with minimum imperfection. Figure (3) shows significantly that increasing the imperfection of slabs with

similar thicknesses reduces the maximum dynamic deflection.

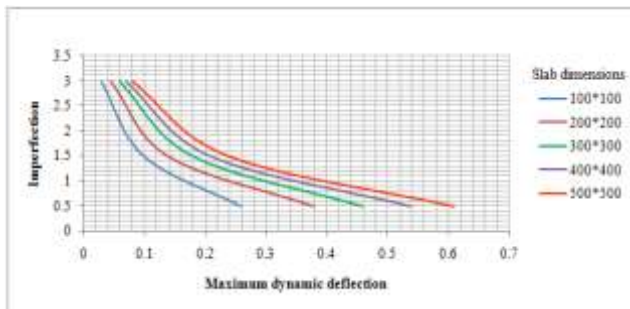


Fig. 3: Relation between Maximum Dynamic Deflection and Imperfection

The previous facts will satisfy any other dimensions but no governing relation of the optimum dimensions can be obtained, classical searching among the variables or mathematical optimization method is the only way for solving the problem.

Classical researching through the equations (2, 3, 4) has been done not only to get more details about the slabs behavior but also to investigate an inflect or critical points, those points might be informative to specify the optimum locations. The equations by considering the variables (wd, a, h) have been drawn with open scale by using MATLAB language as shown in Figures (4),(5) and (6).

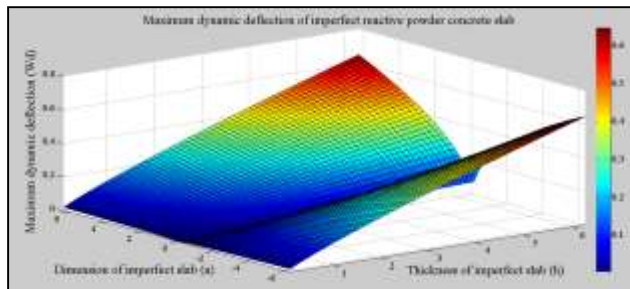


Fig. 4: Surface Drawing of Maximum Dynamic Deflection of Equation ( $c/h=0.167$ )

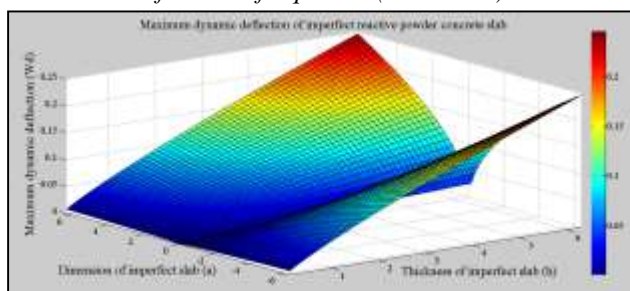


Fig. 5: Surface Drawing of Maximum Dynamic Deflection of Equation ( $c/h=0.5$ )

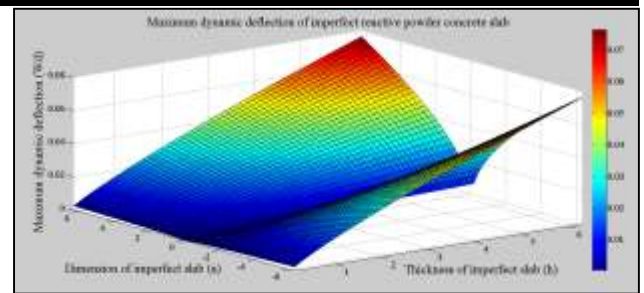


Fig. 6: Surface Drawing of Maximum Dynamic Deflection of Equation ( $c/h=1$ )

According to the previous figures, the main points that have been obtained are :

- In one figure, changing the lengths in small thicknesses don't affected significantly on the maximum dynamic deflection while it has significant effect in large thicknesses.
- In one figure, the effect of changing the thickness in short lengths is more than the effectiveness of point (a), that effect increases by increasing lengths.
- Comparing the figures by each other's shows that imperfection is highly affected on maximum dynamic deflection, the maximum dynamic deflection has been reduced from (0.8) of ( $c/h=0.167$ ) down to (0.08) of ( $c/h=1$ )

According to the previous results, the optimization has to be included three variables (thickness, length, imperfection) because the thickness and imperfection at mid span have obviously effectiveness on problem whereas the length (dimensions of structure) will be variable by according to the site plan.

**3.2 OPTIMIZATION METHOD OF IMPERFECT SLABS:** Some of optimization problems can be solved without constraints and other problems can't solved unless restricting the problem by constraints. It is obviously from the Figures (4,5,6) that the current problem has to be restricted to find the best solution. The constraints of current study represented by restricting the slab to behave as a thin plate according to Kirchhoff theory where the minimum thickness to length should not be more than ( $a/20$ ), on the other hand, to simplify the solution, thickness of slab will be taken according to ACI-code where it has to be not more than ( $a/33$ ).

Regarding the impact force ( $F_m$ ) and flexural rigidity ( $D$ ) have not affected on optimization problem, hence, they were taken according to the data of imperfect slabs of Shhatha et. al.

$$F_m = \text{Impact force} = 11870 \text{ N}$$

$$D = \text{flexural rigidity} = \frac{Eh^3}{12} \text{ where } E = 57 \times 10^9 \frac{\text{N}}{\text{m}^2}$$

The present study adopted to program Modified Hooke-Jeeves method by using Visual Basic language, the program will investigate the optimal case in three cases as shown below:

### One variable

Here, the design variables will be just the thickness (h), the length of slab (a) and mid-span imperfection (c) will be fixed due to architectural reason or any other limitations. Objective function, constraints and design variables have been clarified in Table (2) whereas Figure (7) shows the results of Visual Basic program of optimization technique:

Table.2: Optimum Imperfection-One Variable

Minimum Dynamic Deflection	
Objective function	$W_d = 0.0771 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1305}$ $W_d = 0.0289 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1383}$ $W_d = 0.0087 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1521}$
Constraints	Minimum thickness to length , Minimum thickness
Design variables	Thickness of imperfect slab (h)
Method of optimization	Modified Hooke-Jeeves

Initial value = 2.70280199976216E-03 x 1=0.04 Exploration step = 2.35555644503064E-03 x 1=0.03 Pattern move = 1.94053684743026E-03 x 1=0.02 Exploration step = 1.94053684743026E-03 x 1=0.02 Pattern move = 1E+30 x 1=0.01 Exploration step = 1.94053684743026E-03 x 1=0.02 Base point = 1.94053684743026E-03 x 1=0.02 Exploration step = 1.94053684743026E-03 x 1=0.02 Exploration step = 1.8935369183012E-03 x 1=0.019 Pattern move = 1.84522705283669E-03 x 1=0.018	Exploration step = 1.84522705283669E-03 x 1=0.018 Pattern move = 1E+30 x 1=0.017 Exploration step = 1.84522705283669E-03 x 1=0.018 Base point = 1.84522705283669E-03 x 1=0.018 Exploration step = 1.84522705283669E-03 x 1=0.018 Exploration step = 1.84031982463957E-03 x 1=0.0179 Pattern move = 1.83539826499756E-03 x 1=0.0178 Exploration step = 1.83539826499756E-03 x 1=0.0178 Pattern move = 1E+30 x 1=0.0177 Exploration step = 1.83539826499756E-03 x 1=0.0178
Final result (Optimum imperfection)	
*****	
x 1= 0.022	
Minimum of objective function = 2.26994950031884E-03	
Number of function evaluation = 60	

Fig. 7: Results of Visual Basic Program – One Variable

Different cases of imperfection and length have been taken into consideration and it is noted that the thickness tends to be close to constraint of minimum thickness, hence, in such a problem of one variable (thickness), minimum thickness will be the optimal solution.

### Two variables

The design variables will be the thickness of imperfect slab (h) and the length of slab (a), mid-span imperfection (c) will be fixed due to architectural reason or any other

limitations. Objective function, constraints and design variables have been clarified in Table (3) whereas Figure (8) shows the results of Visual Basic program of optimization technique :

Table.3: Optimum Imperfection-Two Variables

Minimum Dynamic Deflection	
Objective function	$W_d = 0.0771 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1305}$ $W_d = 0.0289 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1383}$ $W_d = 0.0087 h \left( \frac{F_m \cdot a^4}{D h} \right)^{0.1521}$
Constraints	Minimum thickness to length , Minimum thickness
Design variables	Thickness of imperfect slab (h), length of imperfect slab (a)
Method of optimization	Modified Hooke-Jeeves

Initial value = 2.70280199976216E-03 x 1=0.04 x 2=0.8 Exploration step = 2.35555644503064E-03 x 1=0.03 x 2=0.8 Exploration step = 2.34014023634781E-03 x 1=0.03 x 2=0.79 Pattern move = 1.91505963761523E-03 x 1=0.02 x 2=0.78 Exploration step = 1.91505963761523E-03 x 1=0.02 x 2=0.78 Exploration step = 1.9022039491901E-03 x 1=0.02 x 2=0.77	Pattern move = 1E+30 x 1=0.01 x 2=0.75 Exploration step = 1.87625084107423E-03 x 1=0.02 x 2=0.75 Exploration step = 1.86315024584257E-03 x 1=0.02 x 2=0.74 Pattern move = 1.8233321026263E-03 x 1=0.02 x 2=0.71 Exploration step = 1.8233321026263E-03 x 1=0.02 x 2=0.71 Exploration step = 1.80988131865363E-03 x 1=0.02 x 2=0.7
Final result (Optimum imperfection)	
*****	
x 1= 9.99999999999999E-05	
x 2= 2.09999999999999E-03	
Minimum of objective function = 6.93130571385325E-06	
Number of function evaluation = 546	

Fig. 8: Results of Visual Basic Program – Two Variables

It is noted that both of thickness and length of imperfect slab tend to be in a minimum as possible, the ratio of decreasing of thickness and length seem to be similar. The best ratio of (h/a) that has been obtained was about (0.0476), hence, in such a problem, the solution will be according to the following steps:

$$\text{Optimum ratio of } \frac{h}{a} = 0.0476$$

Apply (a) by according to requirements  $\rightarrow h$

The values of (a, h) will be the optimum dimensions in which the maximum dynamic deflection will be minimized as possible.

### Three variables

In such a case, the problem will be general, the design variables will be the thickness (h), the length of slab (a) and mid-span imperfection (c). Regression of the three equations (2,3,4) has been done in order to produce a

relation among the three variables as shown in Table (4), Figure (9) shows the results of Visual Basic program of optimization technique.

Table.4: Optimum Imperfection-Three Variables

Minimum Dynamic Deflection	
Objective function	$W_d = 0.0421 + 0.00113L + 0.0468h + 0.1025C$
Constraints	Minimum thickness to length , Minimum thickness
Design variables	Thickness of imperfect slab (h), length of imperfect slab (a), Imperfection (c)
Method of optimization	Modified Hooke-Jeeves

Initial value = 0.0122 x 1=4 x 2=80 x 3=3 Exploration step = 0.0121532 x 1=3.999 x 2=80 x 3=3 Exploration step = 0.01215207 x 1=3.999 x 2=79.999 x 3=3 Exploration step = 1.20495700000001E-02 x 1=3.999 x 2=79.999 x 3=3.001 Pattern move = 0.01189914 x 1=3.998 x 2=79.998 x 3=3.002 Final result (Optimum imperfection) ***** x 1= 3.91900000000001 x 2= 79.9189999999996 x 3= 3.08099999999999 Minimum of objective function = 1.5170000000897E-05 Number of function evaluation = 156	Exploration step = 1.18523400000001E-02 x 1=3.997 x 2=79.998 x 3=3.002 Exploration step = 1.18512100000001E-02 x 1=3.997 x 2=79.997 x 3=3.002 Exploration step = 1.17487100000001E-02 x 1=3.997 x 2=79.997 x 3=3.003 Pattern move = 1.14478500000001E-02 x 1=3.995 x 2=79.995 x 3=3.005 Exploration step = 1.14010500000001E-02 x 1=3.994 x 2=79.995 x 3=3.005
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Fig. 9: Results of Visual Basic Program –Three Variables

The best ratio of (h/a) that has been obtained was about (0.049) whereas the best ratio of (c/h) was about (0.785). In such a problem, the solution will be according to the following steps:

$$\text{Optimum ratio of } \frac{h}{a} = 0.049$$

$$\text{Optimum ratio of } \frac{c}{h} = 0.785$$

Apply (a) by according to requirements  $\rightarrow h, c$

The values of (a, h, c) will be the optimum dimensions in which the maximum dynamic deflection will be minimized as possible. Shhatha et. al. used specimen (80\*80)cm with (4)cm thickness and (2)cm imperfection whereas the optimum dimensions according to the previous results should be (80\*80)cm with (3.92)cm thickness and (3.077)cm imperfection, in such a case, the maximum dynamic deflection of optimum dimensions has been decreased about (3.7) times than that of Shhatha et. al.

#### IV. APPLICABLE DESIGN TABLES

According to the optimum relations that have been obtained from three variables case, many cases for various dimensions of imperfect slab can be prepared, Table (5) shows some of cases started from imperfect slab (100\*100)cm which can be used as a model for test up to imperfect slab (500\*500)cm that can be used in a site.

Table.5: Optimum Imperfection of Various Dimensions of Imperfect slabs

Optimum Thickness (cm)	Optimum Dimensions of Imperfect Slabs (cm)				
	100*100	200*200	300*300	400*400	500*500
4.9	3.84 cm				
9.8		7.69 cm			
14.7			11.53 cm		
19.6				15.38 cm	
24.5					19.23 cm

\*Shaded cells refer to the optimum imperfection

#### V. CONCLUSIONS

- Minimum thickness of imperfect reactive powder concrete slab under impact load is considered the optimal solution for problems have one variable (thickness effectiveness).
- The optimal ratio of imperfect reactive powder concrete slab thickness to one of its dimension is equal to (0.0476) for problems that have two variables (thickness and length effectiveness).
- In three variables problem, effect of thickness-length-imperfection, the optimal ratio of imperfect reactive powder concrete slab thickness to the one of its dimension is equal to (0.049) whereas the optimal ratio of imperfection to the thickness of slab is equal to (0.785)

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